Critical behaviour of time-dependent Green functions of boson gas from renormalization group I

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Outline

Dynamics of Bose-Einstein condensation

Green functions of QFT

Interaction representation

Functional representation of perturbation theory

Unusual divergences

Introduction

Discovery of BEC in alkali vapours (Cornell, Ketterle, Wiemann 1995, Nobel prize 2001) boosted activity in low-temperature quantum systems.

Magneto-optical trap used – inhomogeneous system. Thermodynamic limit at vanishing force constants restores homogeneity.

Description of dynamics through Gross-Pitaevskii equation for the expectation value $\psi(t,\mathbf{x})$ of the field operator $\hat{\psi}(t,\mathbf{x})$:

$$i\hbar\frac{\partial\psi(t,\mathbf{x})}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + U(\mathbf{x})\right]\psi(t,\mathbf{x}) + g|\psi(t,\mathbf{x})|^2\psi(t,\mathbf{x})$$

generated by the local-interaction Hamilton operator

$$\hat{H} = \int d\mathbf{x}\left\{\hat{\psi}^+(t,\mathbf{x})\left[-\frac{\hbar^2\nabla^2}{2m} + U(\mathbf{x})\right]\hat{\psi}(t,\mathbf{x}) + \frac{g}{2}\hat{\psi}^{+2}(t,\mathbf{x})\hat{\psi}^2(t,\mathbf{x})\right\}.$$

Expectation values of time-ordered products (T products) of field operators are basic tools in applications of QFT:

$$G_n(x_1, x_2, \ldots x_n) = \mathrm{Tr}\left\{\hat{\rho}\, T\left[\hat{\varphi}_H(x_1)\hat{\varphi}_H(x_2)\cdots\hat{\varphi}_H(x_n)\right]\right\} .$$

Operators are written in the Heisenberg picture (I will keep track of the initial time instant t_0 for a while)

$$\hat{\varphi}_H(x) \equiv \hat{\varphi}_H(t, \mathbf{x}) = \exp\left[i(t-t_0)(\hat{H}-\mu\hat{N})/\hbar\right] \times \hat{\varphi}(\mathbf{x}) \exp\left[-i(t-t_0)(\hat{H}-\mu\hat{N})/\hbar\right] .$$

Green functions of particle physics are generated by the density operator of the physical ground state

$$\hat{\rho} = |\,0\,)(\,0\,| .$$

Temperature Green functions

In condensed matter and solid state at $T > 0$ the grand-canonical density operator is used

$$\hat{\rho} = \frac{\exp\left[-\beta(\hat{H} - \mu\hat{N})\right]}{\mathrm{Tr}\exp\left[-\beta(\hat{H} - \mu\hat{N})\right]} \,.$$

Simple evaluation rules with Euclidean evolution (formally imaginary time: $t \to -it$), i.e.

$$\hat{\varphi}_{HE}(t, \mathbf{x}) = \exp\left[(t - t_0)(\hat{H} - \mu\hat{N})/\hbar\right]\hat{\varphi}(\mathbf{x})\exp\left[-(t - t_0)(\hat{H} - \mu\hat{N})/\hbar\right] \,.$$

Evolution must be restricted to finite "time" interval: usually $0 \leq t \leq \beta\hbar$.
Periodic or antiperiodic BC in time: frequency integrals replaced by sums.
Otherwise Feynman rules similar to those at $T = 0$.

Time-dependent Green functions at finite temperature

Statistical averaging of genuine dynamic quantities generates kinetic description.

In condensed matter and solid state at $T > 0$ the grand-canonical density operator $\hat{\rho} = \exp\left[-\beta(\hat{H} - \mu\hat{N})\right] / \mathrm{Tr}\exp\left[-\beta(\hat{H} - \mu\hat{N})\right]$ defines time-dependent Green functions at finite temperature (GF@FT).

Due to complications in evaluation (even in perturbation theory) these are often replaced by temperature Green functions.

Spectral representation reveals the connection between the two through analytic continuation.

Kinetic theory of, say, low-emperature alkali vapours calls for genuine time evolution, i.e. GF@FT.

Physical quantities from Green functions

In non-relativistic condensed matter $\hat{\varphi}$ is a pair of fields $\hat{\varphi}(t, \mathbf{x}) = \left(\hat{\psi}, \hat{\psi}^{+}\right)$ and the one-particle Green function is defined as (bosons here)

$$G_2(t_1, \mathbf{x}_1, t_2, \mathbf{x}_2) = \mathrm{Tr}\left\{\hat{\rho}\, T\left[\hat{\psi}_H(t_1, \mathbf{x}_1)\hat{\psi}_H^{+}(t_2, \mathbf{x}_2)\right]\right\} =$$
$$\theta(t_1 - t_2)\mathrm{Tr}\left[\hat{\rho}\,\hat{\psi}_H(t_1, \mathbf{x}_1)\hat{\psi}_H^{+}(t_2, \mathbf{x}_)\right]$$
$$+\,\theta(t_2 - t_1)\mathrm{Tr}\left[\hat{\rho}\,\hat{\psi}_H^{+}(t_2, \mathbf{x}_2)\hat{\psi}_H(t_1, \mathbf{x}_1)\right]\ .$$

Connection to physics through one-particle density matrix

$$\rho(t_1, \mathbf{x}_1, \mathbf{x}_2) = \lim_{t_2 \to t_1 + 0} G_2(t_1, \mathbf{x}_1, t_2, \mathbf{x}_2)$$

Dirac picture

In perturbation theory time evolution is generated by the free Hamilton operator $\hat{H}_0$ (including number operator in grand canonical ensemble):

$$\hat{\varphi}(t,\mathbf{x}) = \exp\left[i(t-t_0)(\hat{H}_0 - \mu\hat{N})/\hbar\right]\hat{\varphi}(\mathbf{x})\exp\left[-i(t-t_0)(\hat{H}_0 - \mu\hat{N})/\hbar\right] ,$$

where $\hat{H}_0 = \hat{H} - \hat{V}$. Perturbation expansion generated by evolution operator

$$\hat{U}(t,t') = \exp\left[i(t-t_0)(\hat{H}_0 - \mu\hat{N})/\hbar\right]\exp\left[-i(t-t')(\hat{H} - \mu\hat{N})/\hbar\right] \times \exp\left[-i(t'-t_0)(\hat{H}_0 - \mu\hat{N})/\hbar\right] .$$

The evolution operator is not translation invariant! Evolution equations

$$\frac{\partial\hat{U}(t,t')}{\partial t} = -\frac{i}{\hbar}\hat{V}(t)\hat{U}(t,t') , \qquad \frac{\partial\hat{U}(t,t')}{\partial t'} = \frac{i}{\hbar}\hat{U}(t,t')\hat{V}(t') .$$

Evolution operator

Iterative solution yields either chronological exponential

$$\hat{U}(t,t') = 1 - \frac{i}{\hbar}\int_{t'}^{t}\hat{V}(u)du + \left(\frac{i}{\hbar}\right)^2 \int_{t'}^{t}\hat{V}(u_2)\int_{t'}^{u_2}\hat{V}(u_1)du_2du_1 + \dots$$
$$= T\exp\left[-\frac{i}{\hbar}\int_{t'}^{t}V_n(\hat{\varphi}(u))\,du\right]$$

convenient at $t > t'$, or **antichronological** exponential $(t < t')$

$$\hat{U}(t,t') = 1 + \frac{i}{\hbar}\int_{t}^{t'}\hat{V}(u)du + \left(\frac{i}{\hbar}\right)^2 \int_{t}^{t'}\hat{V}(u_1)\int_{t}^{u_2}\hat{V}(u_2)du_2du_1 + \dots$$
$$= \widetilde{T}\exp\left[\frac{i}{\hbar}\int_{t}^{t'}V_n(\hat{\varphi}(u))\,du\right]$$

where $V_n(\hat{\varphi})$ – normal form of operator functional: $\hat{V} = V_n(\hat{\varphi}) = N\left[V_n(\hat{\varphi})\right]$.

Fundamental statement

Starting point of perturbation theory of any GF:

$$G_n(x_1, x_2, \ldots x_n) = \mathrm{Tr}\left\{\hat\rho\, T\left[\hat\varphi_H(x_1)\hat\varphi_H(x_2)\cdots\hat\varphi_H(x_n)\right]\right\}$$
$$= \mathrm{Tr}\left\{\hat\rho\,\hat U\left(0, t_f\right) T\left[\hat\varphi(x_1)\hat\varphi(x_2)\cdots\hat\varphi(x_n)\hat U\left(t_f, t_i\right)\right]\hat U\left(t_i, 0\right)\right\}$$

where $t_i \le t_l \le t_f \quad \forall l = 1, 2, \ldots n$.

All four operators are presented as ordered products and the expression fused to a single normal product by Wick's theorem.

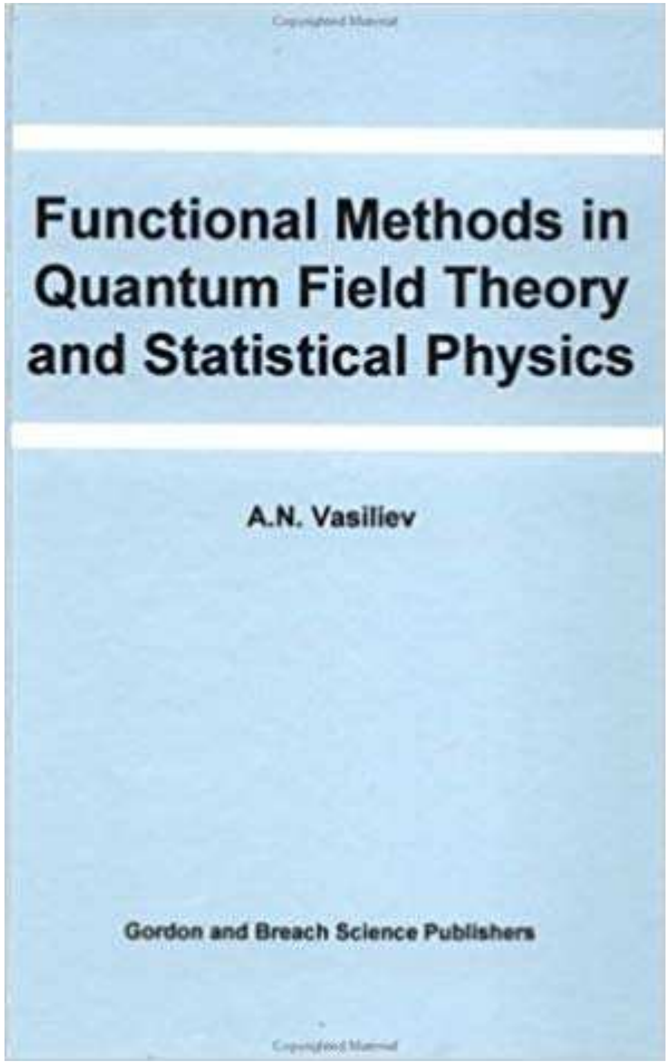
Evolution operators outside the T product difficult to handle.

In particle physics Gell-Mann Low theorem yields evolution of the physical ground state to the free ground state.

In temperature GF the "time" limits are chosen such that $\hat\rho\,\hat U\left(0, t_f\right) = 1$ and $\hat U\left(t_i, 0\right) = 1$

Where to find the functional stuff?

The fundamental statement and details of the functional approach may be found in the classic monograph



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The Russian original is available in the web in djvu format.

Green functions at zero temperature

At zero temperature Gell-Mann Low theorem makes things simpler, the ground state evolves to that ot the free system $|\,0\,\rangle$:

$$\hat{U}(t_f, 0)|\,0) \underset{t_f \to \infty}{\sim} \beta(t_f)|\,0\,\rangle\,, \qquad \hat{U}(t_i, 0)|\,0) \underset{t_i \to -\infty}{\sim} \alpha(t_i)|\,0\,\rangle\,.$$

Green functions become expectation values of operator products in the ground state of the free system

$$\begin{aligned}&(\,0\,|T\,[\hat{\varphi}_H(x_1)\hat{\varphi}_H(x_2)\cdots\hat{\varphi}_H(x_n)]\,|\,0\,) \\ &\quad = \lim_{\substack{t_f \to \infty \\ t_i \to -\infty}} \frac{\left\langle 0 \,\middle|\, T\left[\hat{\varphi}(x_1)\cdots\hat{\varphi}(x_n)\hat{U}(t_f, t_i)\right] \,\middle|\, 0 \right\rangle}{\left\langle 0 \,\middle|\, \hat{U}(t_f, t_i) \,\middle|\, 0 \right\rangle}\,.\end{aligned}$$

Textbook Feynman rules of perturbation theory are for these.

Time-dependent Green functions at $T > 0$

Introduce T exponents in all terms of the fundamental statement:

$$\mathrm{Tr}\left\{\hat{\rho}\,T\left[\hat{\varphi}_H(x_1)\cdots\hat{\varphi}_H(x_n)\right]\right\} = \frac{Z_0}{Z_G}\,\mathrm{Tr}\Bigg(\hat{\rho}_0 T\exp\left[\frac{1}{\hbar}\,S_V(\hbar\beta,0,\hat{\varphi}_E)\right]$$
$$\times\,\widetilde{T}\exp\left[-\frac{i}{\hbar}\,S_V(t_f,t_0,\hat{\varphi})\right]T\left\{\hat{\varphi}(x_1)\cdots\hat{\varphi}(x_n)\,\exp\left[\frac{i}{\hbar}\,S_V(t_f,t_i,\hat{\varphi})\right]\right\}$$
$$\times\,\widetilde{T}\exp\left[-\frac{i}{\hbar}\,S_V(t_0,t_i,\hat{\varphi})\right]\Bigg)\,,$$

where the shorthand $S_V(t',t,\hat{\varphi}) \equiv= -\int_{t'}^{t} V_n(\hat{\varphi}(u))\,du$ has been used.

Invariance of the trace allows to calculate it in the basis spanned by eigenfunctions of the free Hamilton operator. Therefore, Wick theorem allows to calculate expected values with finite time parameters $t_f > t_0 > t_i$.

Wick fusion to a single normal product

Due to a Wick theorem, a single normal product is brought about

$$\mathrm{Tr}\left\{\hat\rho\, T\left[\hat\varphi_H(x_1)\cdots\hat\varphi_H(x_n)\right]\right\} = \frac{Z_0}{Z_G}\mathrm{Tr}\left(\hat\rho_0 N\left\{\exp\left(\frac{1}{2}\sum_{l=1}^{4}\frac{\delta}{\delta\varphi_l}\Delta_{ll}\frac{\delta}{\delta\varphi_l}\right.\right.\right.$$
$$\left. + \sum_{k<l}\frac{\delta}{\delta\varphi_k}n_{kl}\frac{\delta}{\delta\varphi_l}\right)\varphi_3(x_1)\cdots\varphi_3(x_n)\exp\left[\frac{1}{\hbar}S_V(\hbar\beta,0,\varphi_1)\right.$$
$$\left.\left.\left. - \frac{i}{\hbar}S_V(t_f,t_0,\varphi_2) + \frac{i}{\hbar}S_V(t_f,t_i,\varphi_3) - \frac{i}{\hbar}S_V(t_0,t_i,\varphi_4)\right]\right\}\right|_{\substack{\varphi_{1,2,4}=\hat\varphi\\ \varphi_3=\hat\varphi_E}}\right).$$

Contractions: Δ_{11}, Δ_{33} – chronological (differ by operator evolution rules)
$\Delta(x,x') = T\left[\hat\varphi(x)\hat\varphi(x')\right] - N\left[\hat\varphi(x)\hat\varphi(x')\right]$, $\Delta_{22}=\Delta_{44}$ – antichronological
$\widetilde{\Delta}(x,x') = \tilde{T}\left[\hat\varphi(x)\hat\varphi(x')\right] - N\left[\hat\varphi(x)\hat\varphi(x')\right]$, simple contractions by the rule
$n(x,x') = \hat\varphi(x)\hat\varphi(x') - N\left[\hat\varphi(x)\hat\varphi(x')\right]$ (two evolution patterns).

Grand canonical expected value of an operator functional

The free grand-canonical expectation value of the normal product is calculated with the aid of the relation

$$\frac{\mathrm{Tr}\left\{\exp\left[-\beta\hat{H}_0-\mu\hat{N}\right]N[F(\hat{\varphi})]\right\}}{\mathrm{Tr}\exp\left[-\beta\hat{H}_0-\mu\hat{N}\right]}=\exp\left(\frac{1}{2}\frac{\delta}{\delta\varphi}d\frac{\delta}{\delta\varphi}\right)F(\varphi)\Big|_{\varphi=0},$$

with the thermal contraction

$$d(x,x')=\frac{\mathrm{Tr}\left\{\exp\left[-\beta\hat{H}_0-\mu\hat{N}\right]N\left[\hat{\varphi}(x)\hat{\varphi}(x')\right]\right\}}{\mathrm{Tr}\exp\left[-\beta\hat{H}_0-\mu\hat{N}\right]}.$$

In the present case of four fields we have a 4×4 contraction matrix.

Functional representation of GF@FT

Now we have got rid of all the operators:

$$
\begin{aligned}
\mathrm{Tr}\left\{\hat\rho\, T\left[\hat\varphi_H(x_1)\cdots\hat\varphi_H(x_n)\right]\right\} = \frac{Z_0}{Z_G}\Bigg\{\exp\Bigg(\frac{1}{2}\sum_{l=1}^{4}\frac{\delta}{\delta\varphi_l}\Delta_{ll}\frac{\delta}{\delta\varphi_l} \\
+\sum_{k<l}\frac{\delta}{\delta\varphi_k}n_{kl}\frac{\delta}{\delta\varphi_l}+\sum_{k,l=1}^{4}\frac{\delta}{\delta\varphi_k}d_{kl}\frac{\delta}{\delta\varphi_l}\Bigg)\varphi_3(x_1)\cdots\varphi_3(x_n)\exp\Bigg[\frac{1}{\hbar}S_V(\hbar\beta,0,\varphi_1) \\
-\frac{i}{\hbar}S_V(t_f,t_0,\varphi_2)+\frac{i}{\hbar}S_V(t_f,t_i,\varphi_3)-\frac{i}{\hbar}S_V(t_0,t_i,\varphi_4)\Bigg]\Bigg\}\Bigg|_{\varphi_i=0},
\end{aligned}
$$

Standard Feynman rules produced by functional derivatives. Left side independent of time parameters $t_f > t_0 > t_i$, but they show in perturbation expansion generated by the right side.

Propagators in GF@QFT

For further analysis, specify model. Take the simplest nonrelativistic bosonic field theory with the Hamilton operator

$$\hat{H} = \int d\mathbf{x} \left[\hat{\psi}^{+}(t,\mathbf{x}) \left(-\frac{\hbar^2 \nabla^2}{2m} - \mu \right) \hat{\psi}(t,\mathbf{x}) + \frac{g}{2} \hat{\psi}^{+2}(t,\mathbf{x}) \hat{\psi}^{2}(t,\mathbf{x}) \right] .$$

The functional $V_n(\psi^+, \psi)$ is obtained by omitting operator hats.

Chronological contractions are (plane-wave basis)

$$\Delta(t,t';\mathbf{k}) = \theta(t-t') \exp\left[-i\omega(\mathbf{k})(t-t')\right] ,$$

$$\widetilde{\Delta}(t,t';\mathbf{k}) = \theta(t'-t) \exp\left[-i\omega(\mathbf{k})(t-t')\right] ,$$

$$\Delta_E(t,t';\mathbf{k}) = \theta(t-t') \exp\left[-\omega(\mathbf{k})(t-t')\right] ,$$

where $\omega(\mathbf{k}) = \dfrac{\epsilon(\mathbf{k})}{\hbar} = \dfrac{1}{\hbar}\left(\dfrac{\hbar^2 k^2}{2m} - \mu \right)$

Simple and thermal contractions in GF@FT

Both the thermal and simple contractions of Dirac fields are oscillating functions at all times (in chronological contractions the step function cuts off half-axis); thermal contractions

$$
\begin{aligned}
d_{DD}(t,t';\mathbf{k}) &= \exp\left[-i\omega(\mathbf{k})(t-t')\right]\overline{n}(\mathbf{k})\,,\\
d_{EE}(t,t';\mathbf{k}) &= \exp\left[-\omega(\mathbf{k})(t-t')\right]\overline{n}(\mathbf{k})\,,\\
d_{DE}(t,t';\mathbf{k}) &= \exp\left[\omega(\mathbf{k})(-i(t-t_0)+t')\right]\overline{n}(\mathbf{k})\,,\\
d_{ED}(t,t';\mathbf{k}) &= \exp\left[\omega(\mathbf{k})(-t+i(t'-t_0))\right]\overline{n}(\mathbf{k})\,,
\end{aligned}
$$

where $\overline{n}(\mathbf{k})$ is the mean occupation number (bosons!)

$$
\overline{n}(\mathbf{k}) = \frac{1}{\exp\left[\beta\epsilon(\mathbf{k})\right]-1}\,.
$$

Normal contractions are obtained by substitution $\overline{n}(\mathbf{k})\to 1$.

Full propagator matrix of GF@FT

The complete 4×4 matrix of contractions contains different combinations of chronological, simple and thermal contractions

$$\underline{\underline{\Delta}} = \begin{pmatrix} \Delta_E + d_{EE} & n_{ED} + d_{ED} & n_{ED} + d_{ED} & n_{ED} + d_{ED} \\ d_{DE} & \widetilde{\Delta} + d_{DD} & n_{DD} + d_{DD} & n_{DD} + d_{DD} \\ d_{DE} & d_{DD} & \Delta + d_{DD} & n_{DD} + d_{DD} \\ d_{DE} & d_{DD} & d_{DD} & \widetilde{\Delta} + d_{DD} \end{pmatrix}$$

Apart from $\Delta_E + d_{EE}$ all items contain oscillatory term(s): unusual in field theory divergences. For instance, in case of simple-minded Fourier transform

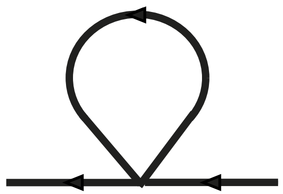
$$d_{DD}(\omega, \mathbf{k}) = 2\pi\, \delta\left[\omega - \omega(\mathbf{k})\right] \overline{n}_{\mathbf{k}}\,, \quad n_{DD}(\omega, \mathbf{k}) = 2\pi\, \delta\left[\omega - \omega(\mathbf{k})\right]$$

with $\omega(\mathbf{k}) = \left(\hbar^2 k^2/2m - \mu\right)/\hbar$ and $\overline{n}(\mathbf{k}) = \left[\exp\left[\beta\epsilon(\mathbf{k})\right] - 1\right]^{-1}$.

Products of propagators with coiciding arguments are ill-defined!

Apparent divergences in limits $t_f \to \infty$, $t_i \to -\infty$

To work on the whole time axis we need these limits. To illustrate problems, consider tadpole contribution to $G(x_1, x_2)$: the contribution with the physical field interaction functional $S_V(t_f, t_i, \varphi_1)$ is



$$= \int_{t_i}^{t_f} dt \, [\Delta(t_1 - t) + d_{DD}(t_1 - t)] \, \Sigma^{(1)}_{\psi_1^+ \psi_1}(0)$$

$$\times [\Delta(t - t_2) + d_{DD}(t - t_2)] = -ig \left[\int d\mathbf{p} \, \overline{n}(\mathbf{p}) \right] \exp\left[-i\omega(\mathbf{k})(t_1 - t_2)\right]$$

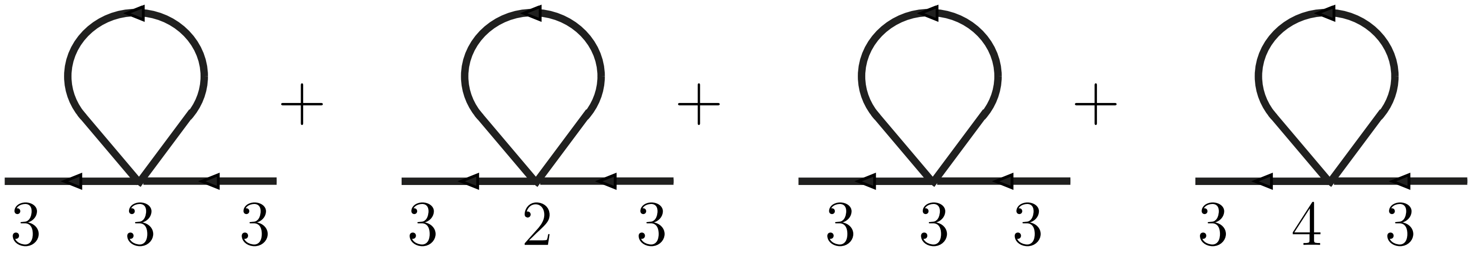
$$\times \left[(t_1 - t_2) + (t_1 - t_i + t_f - t_2)\overline{n}(\mathbf{k}) + (t_f - t_i)\overline{n}^2(\mathbf{k}) \right] .$$

Apparent divergence in both limits $t_f \to \infty$ and $t_i \to -\infty$. Note that in nonrelativistic field theory this graph vanishes.

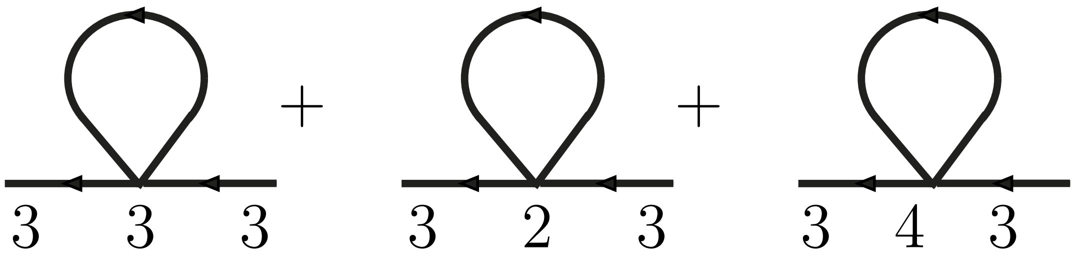
Cancellation of apparent divergences

All contributions to one-loop G_2 (labels are numbers of fields)

$G_2^{(1)}(x_1, x_2) =$



. Divergences in the second and fourth term cancel those of the first:

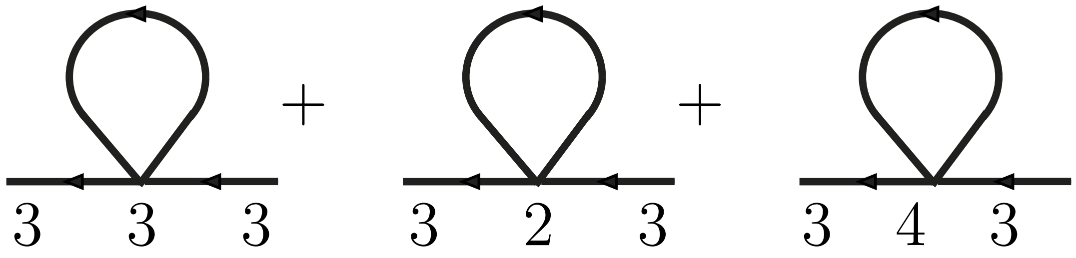


$$= -ig \left[\int d\mathbf{p}\, \overline{n}(\mathbf{p}) \right] \exp\left[-i\omega(\mathbf{k})(t_1 - t_2)\right] (t_1 - t_2) \left[\theta(t_1 - t_2) + \overline{n}(\mathbf{p})\right] .$$

Linear growth $t_1 - t_2$ is unusual, but no divergence. Finite limits in the third.

Renormalization of the chemical potential

Fourier transformation wrt $t_1 - t_2$ yields



$$= -ig \int d\mathbf{p}\, \overline{n}(\mathbf{p}) \left[\frac{-1}{(\omega - \omega(\mathbf{k}) + i\delta)^2} - 2\pi i\, \delta'\left(\omega - \omega(\mathbf{k})\right) \overline{n}(\mathbf{k}) \right]$$

$$= g \int d\mathbf{p}\, \overline{n}(\mathbf{p}) \frac{\partial}{\partial \omega} \left[\frac{i}{\omega - \omega(\mathbf{k}) + i\delta} + 2\pi\, \delta\left(\omega - \omega(\mathbf{k})\right) \overline{n}(\mathbf{k}) \right] .$$

Square brackets of right side – the Fourier transform of $\Delta + d_{DD}$. One-loop contribution is the leading term of an expansion in a shift of the frequency.

Tadpole graphs and the reduced vertex

Standard QFT ordering: closed loops of Δ, $\widetilde{\Delta}$ and Δ_E vanish. The same effect is obtained wrt thermal and simple contractions by introducing the reduced vertex: no tadpoles appear in PT by construction. In the present case $(l = 1, 2, 3, 4)$

$$V_{\mathrm{red}}(\psi_l, \psi_l^+) = \exp\left(\frac{\delta}{\delta\psi_l^+} d \frac{\delta}{\delta\psi_l}\right)\left(\frac{g}{2}\psi_l^{+2}\psi_l^2\right)$$

$$= \frac{g}{2}\psi_l^{+2}\psi_l^2 + 2g\left(\int d\mathbf{p}\,\overline{n}(\mathbf{p})\right)\psi_l^+\psi_l + g\left(\int d\mathbf{p}\,\overline{n}(\mathbf{p})\right)^2 , .$$

The chemical potential is renormalized by the quadratic term and the tadpoles disappear completely.

In higher orders no simple rule of cancellation of divergences: regularization of this divergence is called for.

Pinch singularities of self-energy

Pinch singularities are generated by expansion of full propagators in terms of self-energy:

$$D = \underline{\Delta} + \underline{\Delta}\Sigma\underline{\Delta} + \underline{\Delta}\Sigma\underline{\Delta}\Sigma\underline{\Delta} + \ldots$$

Each correction term contains products of δ functions in frequency with coinciding arguments.

Formal inverse suggests a solution (Dyson equation)

$$D^{-1} = \underline{\Delta}^{-1} - \Sigma\,.$$

Self energy graphs are one-irreducible: no products of identical propagators.

Alas, the inverse of the unregularized propagator is at least ambiguous! Need regularization!

Undamped oscillations

Ordinary quantum-mechanical oscillations are integrable in the momentum space (but this is very inconvenient in perturbation theory), since

$$\int_{-\infty}^{\infty} dk \, \exp\left(-i\,\frac{\hbar k^2}{2m}\right) \propto \int_0^{\infty} \frac{d\epsilon}{\sqrt{\epsilon}} \exp\left(-i\,\frac{\epsilon}{\hbar}\right) \, .$$

In loop integrals of GF@FT undamped oscillations occur, e.g.

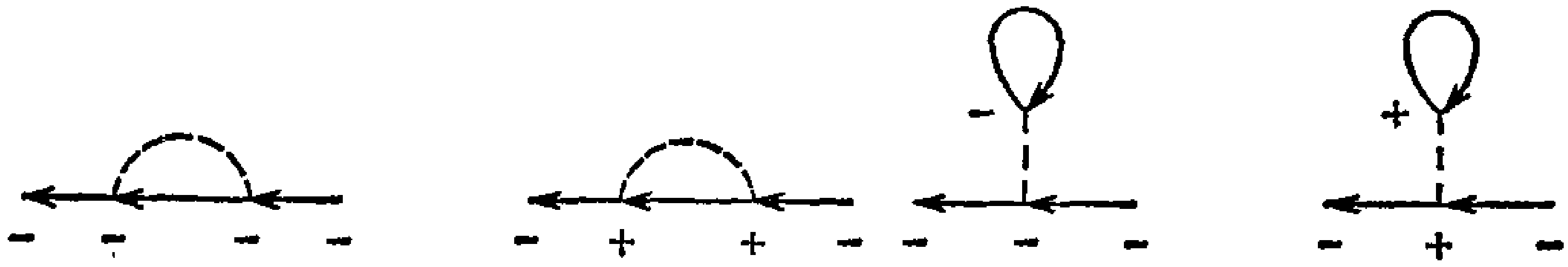
$$-i\frac{\hbar k^2}{2m} + i\frac{\hbar(\mathbf{q}-\mathbf{k})^2}{2m} = \frac{i}{2m\hbar}\left(q^2 - 2\mathbf{k}\cdot\mathbf{q}\right)$$

exponential of which is not integrable over $\mathbf{k}$.

Zero-temperature GF do not have this problem: contractions either retarded or advanced, no pure oscillations.

What did Lifshits and Pitaevskii not say?

In the classic textbook "Physical kinetics" the one-loop graphs are (nonlocal interaction potential, dashed line)



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Nothing about the peculiarities as functions of time (everything in Fourier variables).

As in the majority of literature, what is discussed in detail is Dyson equations, not PT.