Andreev reflection in Weyl semimetals

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Advanced Methods of Modern Theoretical Physics:
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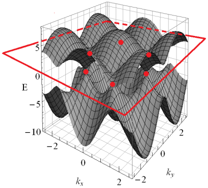
- Topology of Weyl metals and semimetals:
  - non-trivial topology (unlike normal metals and semimetals)
  - this is caused by 3D character of the electronic spectrum, where the Fermi surface (zero energy area) is composed of disconnected sheets, each of them encloses a Weyl node (point of contact between 2 nondegenerate bands)

Andreev reflection in Weyl semimetals

- *examples:*

graphene - point-like Fermi surface



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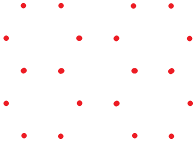
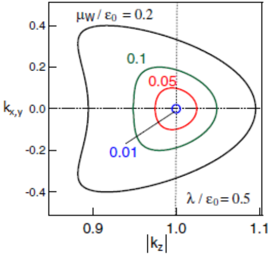
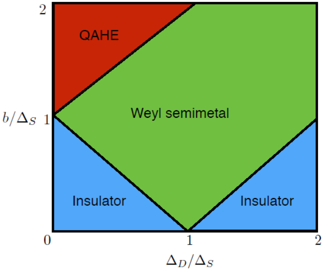
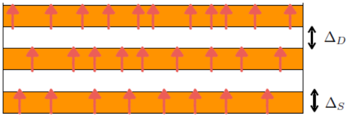


Weyl semimetal - 2D Fermi surface, 1-dimensional cut



- Phase diagram of magnetically doped multilayer structure:





- $\triangle S$ - probability amplitude between top and bottom 2D Dirac surface states
- $\triangle D$ - probability amplitude between neighboring layers
- b - spin splitting due to magnetized impurities
- *QAHE* - topological insulator exhibiting quantum anomalous Hall effect

- Hamiltonian of Weyl semimetals:

$$H = \hbar v_1(\vec{k} - \vec{k_0})\sigma_0 \pm \hbar v_2(\vec{k} - \vec{k_0}) \cdot \vec{\sigma}$$

  - $v_1 < v_2$: WSM of the first kind
  - $v_2 < v_1$: WSM of the second kind

- Weyl semimetals of the first kind
  - coordinate representation for electrons

$$H_W(\vec{r}) = -\frac{\hbar^2}{2m_W}(\nabla^2 + k_0^2)\sigma_z - \mathrm{i}\lambda(\partial_x\sigma_x + \partial_y\sigma_y) - \mu_W\sigma_0$$

  - momentum representation
    - *electrons:*

$$H_W(\vec{k}) = \begin{pmatrix} \epsilon_{\vec{k}} - \mu_W & \lambda(k_x - \mathrm{i}k_y) \\ \lambda(k_x + \mathrm{i}k_y) & -\epsilon_{\vec{k}} - \mu_W \end{pmatrix}$$

    - *holes:*

$$H_W(\vec{k}) = \begin{pmatrix} -\epsilon_{\vec{k}} + \mu_W & \lambda(k_x + \mathrm{i}k_y) \\ \lambda(k_x - \mathrm{i}k_y) & \epsilon_{\vec{k}} + \mu_W \end{pmatrix},$$

  - here,

$$\frac{k_0^2}{\epsilon_0} = \frac{2m_W}{\hbar^2}, \qquad \frac{\epsilon_{\vec{k}}}{\epsilon_0} = \frac{k_x^2 + k_y^2 + k_z^2}{k_0^2} - 1$$

- eigenvalues for electrons

$$\det\left(H_W(\vec{k}) - E\right) = 0 = -(\epsilon_{\vec{k}} - \mu_W - E)(\epsilon_{\vec{k}} + \mu_W + E) - \lambda^2(k_x^2 + k_y^2);$$

$$\lambda^2(k_x^2 + k_y^2) = \lambda^2 p^2 \Rightarrow (\mu_W + E)^2 = (\lambda p)^2 + \epsilon_{\vec{k}}^2 = (E_{\vec{k}}^W)^2 \Rightarrow E = \pm E_{\vec{k}}^W - \mu_W$$

- eigenvectors for electrons

$$\vec{v}_1 = \begin{pmatrix} \alpha_k \\ \beta_k(k_x + \mathrm{i}k_y)/p \end{pmatrix}, \qquad \vec{v}_2 = \begin{pmatrix} -\beta_k(k_x - \mathrm{i}k_y)/p \\ \alpha_k \end{pmatrix},$$

where

$$\alpha_k = \sqrt{\frac{1}{2}\left(1 + \frac{\epsilon_{\vec{k}}}{E_{\vec{k}}^W}\right)}, \qquad \beta_k = \sqrt{\frac{1}{2}\left(1 - \frac{\epsilon_{\vec{k}}}{E_{\vec{k}}^W}\right)};$$

Andreev reflection in Weyl semimetals

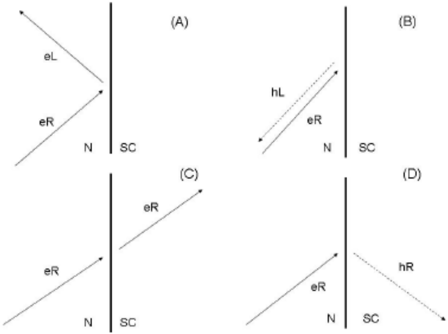
- Physics of WSM quantum dot:

N - normal metal, WSM - Weyl semimetal, SC - superconductor

- non-trivial physics: broken chiral symmetry (chiral anomaly), Fermi arcs
- creation of bound levels due to multiple reflection
- broken time-reversal symmetry

Andreev reflection in Weyl semimetals

- Processes occurring at a contact between superconductor and normal metal junction:



  - speculiar reflection (A)
  - Andreev reflection (B): charge-transfer process by which the normal current in N converts to supercurrent in SC
  - transmission as an electron (C)
  - transmission as a hole (D)

- Hamiltonian of superconductor:

$$H_S(\vec{k}) = \begin{pmatrix} \xi_{\vec{k}}\sigma_0 & \triangle \mathrm{i}\sigma_y \\ -\triangle \mathrm{i}\sigma_y & -\xi^*_{-\vec{k}}\sigma_0 \end{pmatrix}$$

with $\xi_{\vec{k}} = \frac{\hbar^2\vec{k}^2}{2m_S} - \mu_S$

- eigenvalues:

$$E = \pm\sqrt{\left(\frac{\hbar^2\vec{k}^2}{2m_S} - \mu_S\right)^2 + \triangle^2} = \pm\sqrt{\Omega^2 + \triangle^2};$$

Andreev reflection in Weyl semimetals

- eigenfunctions:

$$\psi_{SC}(z>0) = \begin{pmatrix} u_0 C_\uparrow \\ u_0 C_\downarrow \\ -v_0 C_\downarrow \\ v_0 C_\uparrow \end{pmatrix} e^{\mathrm{i}q_e z} + \begin{pmatrix} v_0 D_\downarrow \\ -v_0 D_\uparrow \\ u_0 D_\uparrow \\ u_0 D_\downarrow \end{pmatrix} e^{-\mathrm{i}q_h z},$$

where

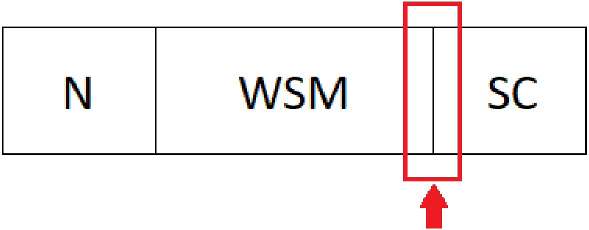
$$u_0 = \sqrt{\frac{1}{2}\left(1+\frac{\Omega}{E}\right)}, \qquad v_0 = \sqrt{\frac{1}{2}\left(1-\frac{\Omega}{E}\right)}$$

with

$$q^{e(h)} = \sqrt{k_F^2 - p^2 + (-)2m_S\Omega/\hbar^2},$$

$$k_F^2 = \frac{k_0^2}{\epsilon_0}\frac{m_S}{m_W}\mu_S$$

- Solution of the system for WSM-SC junction:



- z-direction:

$$\psi_W(z=0) = \psi_{SC}(z=0),$$

$$-\frac{\hbar^2}{2m_W}\begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}\partial_z\psi_W(z=0) = -\frac{\hbar^2}{2m_S}\partial_z\psi_S(z=0) + [V_0\sigma_0 + \vec{V}\cdot\vec{\sigma}]\psi_{SC}(z=0);$$

here, each element of the Pauli matrices is a 2×2 matrix, so

$$V_0\sigma_0 + \vec{V}\cdot\vec{\sigma} = \begin{pmatrix} V_0+V_z & 0 & V_x-\mathrm{i}V_y & 0 \\ 0 & V_0+V_z & 0 & V_x-\mathrm{i}V_y \\ V_x+\mathrm{i}V_y & 0 & V_0-V_z & 0 \\ 0 & V_x+\mathrm{i}V_y & 0 & V_0-V_z \end{pmatrix}$$

Andreev reflection in Weyl semimetals

- the wave function for WSM part

$$\psi_W(z=0) = v_1^e(a^+ + A^+) + v_2^e(a^- + A^-) + v_1^h(b^+ + B^+) + v_2^h(b^- + B^-)$$

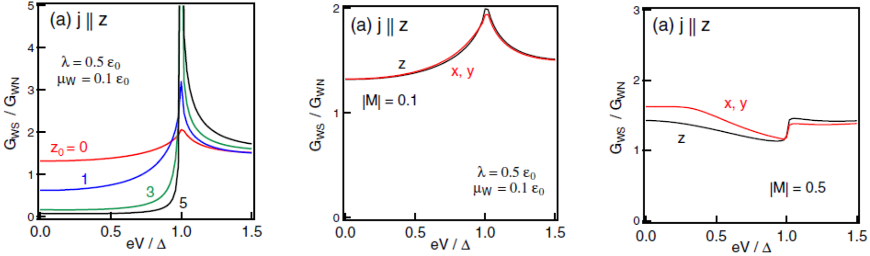
- the coefficients A^+, A^-, B^+, B^- of the outgoing particles and a^+, a^-, b^+, b^- of the incoming particles are connected with the help of the reflection matrix:

$$\begin{pmatrix} A^+ \\ A^- \\ B^+ \\ B^- \end{pmatrix} = \begin{bmatrix} \hat{r}_{ee} & \hat{r}_{eh} \\ \hat{r}_{he} & \hat{r}_{hh} \end{bmatrix} \begin{pmatrix} a^+ \\ a^- \\ b^+ \\ b^- \end{pmatrix};$$

- differential conductance:

$$G_{WS}(eV) = \frac{e^2}{h} \sum_{\vec{\mathbf{P}}} \mathrm{Tr}[\hat{1}_{ee} - \hat{r}_{ee}\hat{r}_{ee}^\dagger + \hat{r}_{he}\hat{r}_{he}^\dagger]\Big|_{E=eV}$$

- z-direction:


(a) j ∥ z
λ = 0.5 ε0
μW = 0.1 ε0
z0 = 0
1
3
5
GWS / GWN
eV / Δ
0.0
0.5
1.0
1.5
(a) j ∥ z
z
x, y
|M| = 0.1
λ = 0.5 ε0
μW = 0.1 ε0
GWS / GWN
eV / Δ
(a) j ∥ z
x, y
z
|M| = 0.5
GWS / GWN
eV / Δ

- Weyl semimetals of the second kind
  - $$H = \begin{pmatrix} H_+\vec{k} - \mu(\vec{r}) & \Delta(\vec{r}) \\ \Delta(\vec{r}) & -H_+\vec{k} + \mu(\vec{r}) \end{pmatrix},$$

    where

    $$H_+(\vec{k}) = \hbar v_1 k_x \sigma_0 + \hbar v_2 \vec{k} \cdot \vec{\sigma},$$

    $\mu(\vec{r}) = \mu$ in the case of Weyl semimetal and U in the case of superconductor, $\Delta(\vec{r}) = 0$ in the case of Weyl semimetal and Δ in the case of superconductor
  - eigenvalues

    $$E_{e\pm} = \hbar v_1 k_x \pm \hbar v_2 k - \mu, \qquad E_{h\pm} = -\hbar v_1 k_x \pm \hbar v_2 k + \mu,$$

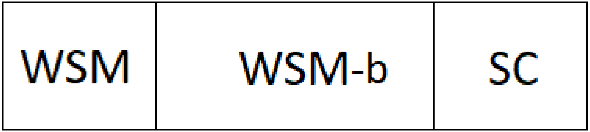
    $$E_{SC} = \sqrt{\Delta^2 + (\hbar v_1 k_x \pm \hbar v_2 k - U)^2}$$
  - eigenvectors for electrons

    $$\psi_{e\pm} = \begin{pmatrix} \pm k_\pm + k_z \\ k_{x\pm} + \mathrm{i}k_y \end{pmatrix} \exp(\mathrm{i}k_{x\pm}x + \mathrm{i}k_y y + \mathrm{i}k_z z)$$
  - eigenvectors for superconductor (2 of them for our purpose)

    $$\psi_{SC\pm} = (e^{\mathrm{i}\beta}, \pm e^{\mathrm{i}\beta}, 1, \pm 1)^T \exp(\mathrm{i}k_{x_{1,2}}x + \mathrm{i}k_y y + \mathrm{i}k_z z - \tau_{1,2}x)$$

Andreev reflection in Weyl semimetals

- influence of the interface barrier:

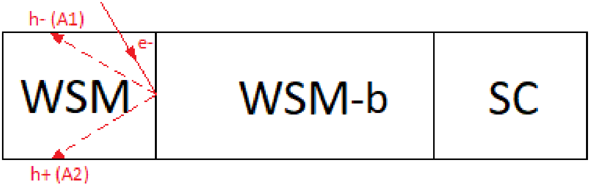


WSM - Weyl semimetal, WSM-b - interface barrier created by WSM, SC - superconductor

here, $\mu(\vec{r}) = \mu$ for WSM and $\mu(\vec{r}) = -V_0$ for WSM-b

Andreev reflection in Weyl semimetals

- double Andreev reflection:



WSM - Weyl semimetal, WSM-b - interface barrier created by WSM, SC - superconductor

here, $\mu(\vec{r}) = \mu$ for WSM and $\mu(\vec{r}) = -V_0$ for WSM-b

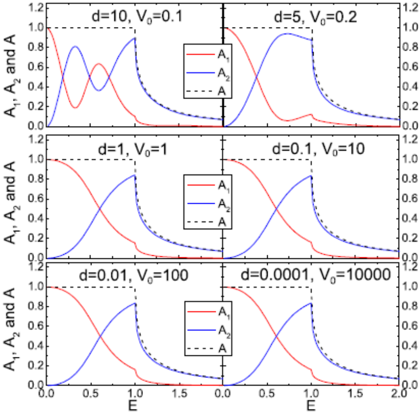
- reflection coefficients:

$$A_1 = \frac{|\psi_{h_-}|J_x|\psi_{h_-}|}{|\psi_{e_-}|J_x|\psi_{e_-}|}|r_1|^2$$

$$A_2 = \frac{|\psi_{h_+}|J_x|\psi_{h_+}|}{|\psi_{e_-}|J_x|\psi_{e_-}|}|r_2|^2$$

Andreev reflection in Weyl semimetals

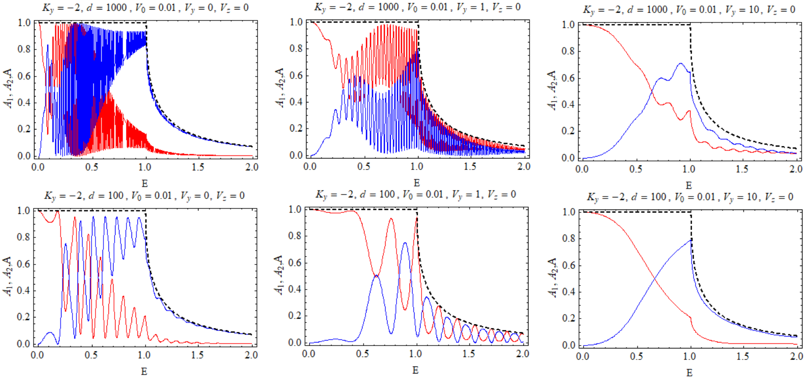
- plots:



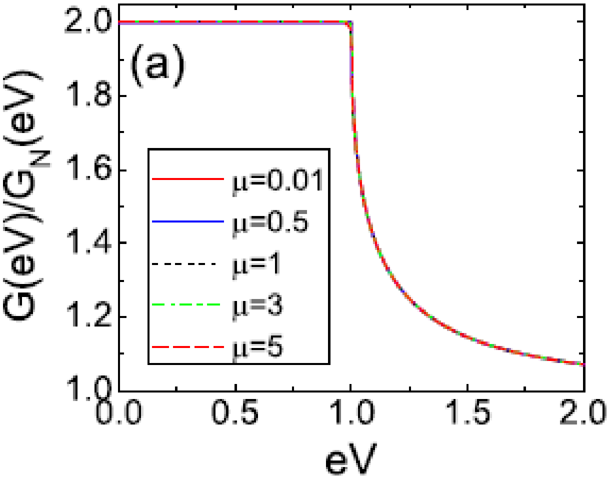
- interesting property: $A_1 + A_2 = 1$ for arbitrary d, V_0 or μ!

Andreev reflection in Weyl semimetals

- non-zero magnetic field:



- conductance: $G(eV)/G_N(0) = \int d\mathrm{k}(1 + A_1 + A_2)$



- Conclusion: the magnetic field does not affect the conductance!
- Proof: in phase of development, needs of evaluation of difficult analytical expressions

- Bibliography

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Thank you for your attention